yields

$$\mathbf{r}_\pm = [1, 0, \pm(1/M_0)], \pm[\cot\theta/M_0) - \csc\theta, b/M_0^2, 0] + [\pm (1/M_0) - \cos\theta)] \times [(0, -\csc\theta, 0, 0, \pm (1/bM_0), 1/b] \quad (5)$$

The corresponding Riemann invariants, $Z_\pm = \mathbf{r}_\pm \cdot \mathbf{P}$, hold along two families of straight transverse characteristics described by

$$dy/dx = \sin\theta (\cos\theta \mp M_0)^{-1} \quad (6a)$$

respectively, which, after integration, becomes

$$y = \sin\theta (\cos\theta \mp M_0)^{-1}x + C_\pm \quad (6b)$$

respectively, with $C_\pm$ being the integration constants or the parameters characterizing two families of straight characteristics, respectively.

It may also be pointed out that in Fig. 1 of Ref. 1 the correct expressions for t, $\tilde{A}_0^2$, and $\tilde{a}_0^2$ should read

$$t = (s^2 - 4a_0^2A_0^2)^{1/2} \qquad \tilde{A}_0^2 = (s - t)/2$$
$$\tilde{a}_0^2 = (s + t)/2 \quad (7)$$

respectively. An obvious misprint appears in (27) of Ref. 1 where the denominator of the integrand should read $\rho - x$.

Reference

[1] Cumberbatch, E., Sarason, L., and Weitzner, H., "Magnetohydrodynamic flow past a thin airfoil," AIAA J. 1, 679 (1963).

Comment on "Steadiness of a Plasmajet"

P. G. Simpkins*
Avco/RAD, Wilmington, Mass.

In a recent article Pfender and Cremers[1] have noted that the inconsistency of temperature measurements obtained in various plasmajet acilities may be attributable to the unsteadiness of the arc column. They quote examples of the well-known blown arc and state that the spectrophotometric results may indicate excessively high temperatures if the arc column extends beyond the anode. The reason for this indication is that spectrographic equipment requires steady state conditions and is not capable of obtaining time resolved measurement. Thus, an unsteady oscillation of the order of 1 kc/sec would not be perceived and only an integrated recording of the spectral line intensities would be obtained.

The purpose of this comment is not to criticize the previous work but merely to point out that Harvey[2] et al., have communicated the existance of this behavior in a note that illustrated the temperature dependence on the source intensity. In that work, Harvey[2] has noted also that the arc instability occurs in nitrogen but not in argon gas, and that the hypothesis of a flexible current carrying conductor (viz., the arc column) is itself an unstable element. Subsequently, Simpkins[3] has shown that a similar instability of the arc column occurs for both helium and hydrogen operation, and that when such an instability occurs in the arc chamber, it manifests itself downstream of a settling chamber and a convergent-divergent nozzle. This result, which confirmed the earlier photodiode findings, was achieved by using an Acmade 35-mm drum camera to monitor the standoff distance of a bow shock wave created when a right circular cylinder was immersed in the test jet. These tests showed that the bow shock wave remained steady for the argon arc but exhibited an oscillatory motion when nitrogen operation was employed. A histogram using a sample of 100 consecutive frames from the drum camera is shown in Fig. 1, where it can be seen that the amplitude of the shock oscillation is some ±25% of the mean distance. Since the drum camera operation was limited to about 4500 frames/sec, only a qualitative comparison could be made with the arc instability frequency; however, within this limitation both sets of data were in agreement with the frequency level of 2 kc/sec.

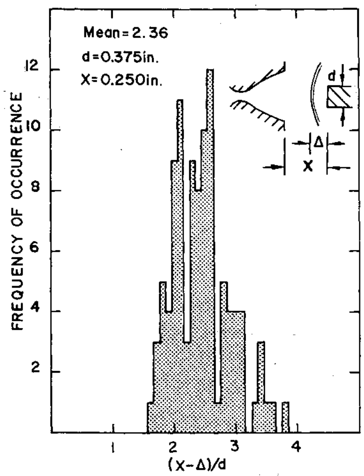

Fig. 1 Histogram of the shock standoff distance variation in an unstable nitrogen arc-jet.

The instability mentioned previously in the arc column is the result of a body force being exerted on the column when a local deformation takes place; the magnitude of which is of order I^2/R (I being the current and R the local radius of curvature). These phenomena have been examined experimentally by Curruthers and Davenport,[4] who have shown that, if the current density in the arc column becomes sufficiently large, an instability is spontaneously established. The possibility of such an effect occuring in a drawn arc facility has been discussed by Harvey et al., who have indications of such an onset when the mass flow rate of the gas has been decreased to a level where the vortex stabilization of the gas is insufficient to restrain the body force of the self-induced magnetic field.

It may be conjectured that the blown arc instability is related to its convective velocity, which is undoubtedly a function of the atomic weight of the gas being used. Therefore, one would expect that the arc column would become more stable as the atomic weight of the gas is increased. However, since there are large gradients and nonequilibrium conditions existing in the arc chamber, this hypothesis probably is oversimplifying the problem. Because of the instabilities described previously, caution must be exercised in interpreting any photographic data, since the time averaged measurements of the flow undoubtedly differ from the equivalent steady state measurements. It should be noted that time resolved temperature measurements can be obtained by coupling two or more photo-multipliers to a high dispersion spectograph.

Received August 18, 1965. The work described herein was carried out while the author was a Research Assistant in the Aeronautics Department of Imperial College, London, England.

* Senior Staff Scientist Aerophysics Department. Member AIAA.

This technique allows time dependent variations of chosen spectral lines to be recorded electronically. Such an investigation is currently being undertaken at Imperial College by Adcock and Plumtree,[5] who are seeking to correlate the temperature measurements with arc instability data.

The conclusions of Pfender and Cremers[1] that the argon arc temperature data are for a steady state condition are probably correct for most facilities; there is evidence,[6] however, that at very low pressures instabilities can occur in argon arcs. The more interesting question relating to the problem still remains unresolved; namely, what is the mechanism that creates arc instabilities in hydrogen, helium, and nitrogen but does not affect the stability of the argon arc column?

References

[1] Pfender, E. and Cremers, C. J., "Steadiness of a plasmajet," AIAA J. **3**, 1345–46 (1965).

[2] Harvey, J. K., Simpkins, P. G., and Adcock, B. D., "Instability of arc columns," AIAA J. **1**, 714–715 (1963).

[3] Simpkins, P. G., "Flow studies in a high enthalpy gas stream," PhD. Thesis, Univ. of London, England (1965).

[4] Carruthers, R. and Davenport, P. A., "Observations of the instability of constricted gaseous discharges," Proc. Phys. Soc. **B**, Ser. 70, 49, 50(1957).

[5] Adcock, B. D. and Plumtree, W. E. G., private communication, Dept. of Aeronautics, Imperial College, London, England (1965).

[6] Bray, K. N. C., private communication, Dept. of Aeronautics and Astronautics, Univ. of Southampton, Hants, England (1964).

Reply by Authors to P. G. Simpkins

E. Pfender*
University of Minnesota, Minneapolis, Minn.

AND

C. J. Cremers†
Georgia Institute of Technology, Atlanta, Ga.

SIMPKINS mentions in his comment that arc "instabilities" did not occur when argon was used as working fluid. This is not in agreement with the paper to which he refers.[1] In this paper the following statement is made: "The results show that two distinct modes of operation occurred: 1) a steadily burning arc, characterized by an almost constant voltage drop between the electrodes; and 2) an unstable arc, recognizable by large amplitude fluctuations in the voltage. When argon was being used as the test gas, both types of operation were observed, the unstable arc occurring only at low mass flow rates."

Investigations in the Heat Transfer Laboratory of the University of Minnesota have revealed the same two modes of operation in an argon atmosphere with a simplified electrode geometry[2] as well as with the F40 plasma torch. The transition from one mode to the other could be predicted by a critical Reynolds number that indicates that the mode of operation is governed largely by gasdynamic effects. The current only had a minor influence on the mode of operation and on the transition from one mode to the other. However, the parameter range investigated in these studies was not wide enough for drawing final conclusions. Therefore, these results are considered to be of a preliminary nature. The arc arrangement certainly is significant for the arc mode. Nevertheless, the same modes of operation at different parameter settings may be observed with different arc geometries.[3] From preliminary tests in other gases (nitrogen, hydrogen), it seems that the mode of operation also depends strongly on the working fluid. With the latter gases, the restriking frequency of the fluctuating arc increases and the parameter range for which the steady mode prevails becomes smaller. It seems conceivable also that the current may have an appreciable influence on the mode of operation in the higher current range where magnetic body forces are of importance.

A distinction should be made between arc instabilities and arc fluctuations. Changes in the arc length accompanied by changes in the arc voltage, which are not related to magnetohydrodynamic instabilities, should be designated as arc fluctuations. Magnetohydrodynamic instabilities to which Simpkins also refers in his comment[4] occur predominantly in high current discharges at low pressures, where the pressure caused by the self magnetic field is of the same order of magnitude or larger than the gas pressure. This situation does not exist in arcs at atmospheric pressure for a current range of several hundred amps.

In conclusion, we agree with Simpkins' statement that the sensitivity of the arc with respect to fluctuations varies with the working fluid. However, the mechanism that triggers these fluctuations seems to be the same for all of the gases that have been investigated. For a given arc geometry the parameter range for which the steady mode prevails may be rather wide for argon,[2] but, nevertheless, the fluctuating mode occurs also. With nitrogen and hydrogen, the parameter range for the steady mode is rather narrow, and, thus, one usually observes the fluctuating mode using these gases as working fluid.

References

[1] Harvey, J. K., Simpkins, P. G., and Adcock, B. D., "Instability of arc columns," AIAA J. 1, 714 (1963).

[2] Pfender, E. and Eckert, E. R. G., "Behavior of an electric arc with superimposed axial flow," *Proceedings of the 1965 Heat Transfer and Fluid Mechanics Institute, 50 (1965)* (Stanford University Press, Stanford, Calif., 1965).

[3] Pfender, E., Wutzke, S. A., Shih, K. T., Eddy, T., and Eckert, E. R. G., "Thermal analysis and study of arc-flow interactions in the anode regime of an electric arc," Heat Transfer Lab., TR-64, University of Minnesota (1965); also NASA CR-54465 (1965).

[4] Carruthers, R. and Davenport, P. A., "Observation of the instability of constricted gaseous discharge," Proc. Phys. Soc., **B**, 49 (1957).

Received October 18, 1965.

* Associate Professor of Mechanical Engineering. Member AIAA.

† Assistant Professor of Mechanical Engineering. Member AIAA.

Errata: "Physics of Meteor Entry"

Mary F. Romig*
The Rand Corporation, Santa Monica, Calif.

[AIAA J. **3**, 385–394 (1965)]

IN this paper† it was stated (on p. 389) that the technique used by Millman and Cook[1] to determine the final mass of a meteor was unclear, because the system appeared to be

Received October 11, 1965.

* Aero-Astronautics Department. Associate Fellow Member AIAA.

† The nomenclature is that of the subject paper.